PAPER ID-310932

Roll No: | | | | | | | | | | | | | |

B TECH
(SEM-III) THEORY EXAMINATION 2020-21
MATHEMATICS-III

Time: 3 Hours ***Total Marks: 70***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 7 = 14**

a. Show that the function $f(z) = x^2 - y^2$ is harmonic function.

b. Define the Poisson distribution with mean and variance.

c. Write the normal equation for the curve $y = \frac{a}{x} + b\sqrt{x}$.

d. Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x . \frac{E\, e^x}{\Delta^2 e^x}$

e. Write the relation between n^{th} divided difference and n^{th} forward difference.

f. What do you mean by initial value problem?

g. Find $Z^{-1}\left(\frac{5}{5z-1}\right)$

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

a. State and prove the Cauchy Integral formula. Also evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = 3/2$,

b. In a partially distributed laboratory record of an analysis of a correlation data, the following result are legible:
Variance $of\ x = 9$
Regression equation: $8x - 10y = 66 = 0, 40x - 18y = 214$.
What were (a) the mean of $x$and y. (b) the standard deviation of y and the coefficient of $x\ and\ y$:

c. Decompose $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ in the form LU, where L is lower triangular matrix and U is upper triangular matrix and hence solve the system of equations:

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10.$$

d. If $F_c(p) = \frac{1}{2} tan^{-1} \frac{2}{p^2}$, then find $f(x)$.

e. Given the initial value problem $\frac{dy}{dx} = x^3 - y^3,\ y(0) = 1$.
Find the numerical solution of differential equation at $x = 0.6\ with\ h = 0.2$ by using Runge-Kutta method of Fourth order.

SECTION C

3. Attempt any *one* part of the following: **7 x 1 = 7**

(a) Evaluate the contour integration: $\int_0^{2\pi} \frac{1}{a + b\sin\theta} d\theta,\ where\ a > |b|$.

(b) Determine the analytic function $f(z) = u + iv$, in terms of whose real part is $e^x(x\cos y - y\sin y)$

4. Attempt any *one* part of the following: **7 x 1 = 7**

(a) Find Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine transform of $\frac{x}{1+x^2}$.

(b) Solve by Z-transform the differential equation $y_{k+2} + 6y_{k+1} + 9y_k = 2^k; (y_0 = y_1 = 0)$.

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5. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) Show that Poisson distribution is a limiting case of Binomial distribution when p is very small and n is very large. Also find the mean and variance of the Poisson distribution.

(b) Find the mean and variance of normal distribution.

6. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) Write the order of convergence of an iterative method. Prove that rate of convergence of Newton Raphson method is quadratic.

(b) State and prove the Lagrange interpolation formula. Find the interpolating polynomial by By Lagrange interpolation formula for the given data.

x	5	6	9	11
y	12	13	14	16

7. **Attempt any *one* part of the following:** **7 x 1 = 7**

(a) State and prove Simpson one third rule and Evaluate $\int_0^2 \frac{1}{x^2+x+1} dx$ using Simpson's rule with eight intervals.

(b) Find x for which y is maximum and find the max value of y

x	1.2	1.3	1.4	1.5	1.6
y	0.9320	0.9636	0.9855	0.9975	0.9996